q_b = heat transferred per unit area and time from vapor within a bubble to bulk liquid
T = temperature; T_w = surface temperature; T_{sat} = saturation temperature
ΔT = temperature difference, $T_w - T_{sat}$
t = time
u_∞ = rise velocity of bubble far from boiling surface
α = thermal diffusivity, $k/\rho c_p$
θ = contact angle between bubble and surface
ρ = density
σ = surface tension

Subscripts

d = at detachment
l = liquid
n = normal (earth) gravity
v = vapor

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Turbulent Disruption of Flocs in Small Particle Size Suspensions

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In the absence of turbulent fluctuations the main effect of a velocity gradient on the floc properties is a rearrangement of particles within the floc producing a more dense floc structure. When the suspension is sufficiently dilute that floc-floc collisions are negligible, the limits on the floc diameter are $(1 + \alpha)^{5/3} < (D_f/D_p) \leqq (1 + \alpha)^2$, where α is the ratio of the volume of fluid immobilized in the floc structure to volume of solids in the floc structure as determined from hindered-settling measurements. These results set an upper limit on the floc size.

Under turbulent flow conditions the principle mechanism leading to floc rupture is pressure differences on opposite sides of the floc which cause bulgy deformation and rupture. The breakup of the floc is resisted by the yield stress τ_y and is promoted by an increase in the energy dissipation per unit mass of fluid ϵ. Because the energy dissipation per unit mass is at a maximum near the pipe wall, the floc size is at a minimum in this same region.

By application of the concepts of local isotropy, the floc size is found to be proportional to $(\tau_y{}^9/\epsilon^5)^{1/2}$ once the turbulent intensity is sufficient to overcome the yield stress. In the wall region the floc diameter is proportional to $(du/dr)^3\ (\tau_y{}^9/\epsilon^8)^{1/2}$.

The hydrodynamic splitting of liquid drops suspended in immiscible liquids has been the subject of many theoretical and experimental studies (*1* to *4*). As a result the factors responsible for a given drop size distribution are rather well defined. In contrast to this the dynamics of the disruption of clumps of flocculated solid particles has received little attention, even though the agglomeration process has been studied in detail (*5, 6*). One reason for this may be the intangible nature of the floc. (A floc may be considered as a loose, irregular, three-dimensional cluster

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of particles in contact in which the original particles can still be recognized). Recent studies have shown that in dilute suspensions flocs or agglomerates may be considered as entities that have reproducible apparent diameters and densities which are a function of rate of shear, concentration, individual particle size, etc. (*7, 8, 9*). Even though a floc cannot be defined as precisely as a liquid drop, under some conditions it may be possible to relate the apparent floc diameters and densities to the characteristics of particular shear flow situations.

The principle differences between an emulsion and a suspension of flocculated particles is that the individual particles making up a floc may be rearranged by motion of the surrounding fluid causing the floc density to be an independent variable. In addition a suspension has no property exactly equivalent to the surface tension. Even if a discrete floc structure is assumed, and the interfacial energy is calculated from the floc geometry and the Derjaguin-Verwey-Overbeek theory (*6*), the values are at least an order of magnitude smaller than interfacial energies observed with most emulsions (*3*). Because of this floc oscillation in general cannot be considered as a disruption mechanism.

The modes of deformation (*2, 10, 11*) in shear flow which may lead to floc disruption are elongation into prolate spheroids under the action of simple shear, bulbuous distortion by irregular dynamic pressures such as those that occur in mechanical agitation or turbulent flow, and for sufficiently large ratios of floc viscosity to suspending medium viscosity the floc might retain a roughly spherical shape while undergoing rotation and particle rearrangement. In the latter case disruption could occur only through the imposition of a shear stress across the floc which would exceed the yield stress. The first two cases might be combined for example as in turbulent shear flow. In this case the action of the velocity gradient may extend the floc, while concurrently the dynamic pressure differences would tend to disrupt the extended floc. Before discussing floc disruption further it is necessary to define the floc characteristics more precisely.

FLOC CHARACTERISTICS

Suspensions of particles having near-colloidal dimensions have been shown to possess non-Newtonian flow characteristics (*6*). Although there is no commonly accepted rheological equation of state which completely describes the laminar flow behavior of flocculated suspensions (*12*), the Bingham plastic model [Equation (1)] has been shown to adequately describe the flow properties of suspensions under many circumstances (*13, 14, 15*), in particular over the shear rate range of 5×10^2 to 5×10^4 sec.$^{-1}$

$$\frac{du}{dr} = \frac{g_c}{\eta}(\tau - \tau_y), \tau > \tau_y \qquad (1a)$$

$$\frac{du}{dr} = 0, \; \tau \leq \tau_y \qquad (1b)$$

Furthermore the two disposable constants τ_y and η were shown to be related to the suspension and particle properties by the empirical expressions

$$\tau_y = 210\, \varphi^3/D_p^2 \qquad (2a)$$

and

$$\eta/\mu = \exp \varphi \left(2.5 + \frac{14}{\sqrt{D_p}}\right) \qquad (2b)$$

when D_p is expressed in microns and τ_y in pounds-force per square feet. Equations (1) and (2) have been shown to apply to a wide variety of suspensions (*9, 14*) and in general will be assumed to apply to the assemblage of particles making up the individual flocs in the present analysis. The purely empirical nature of Equation (2) may be emphasized by noting that Michaels and Bolger (*8*) have reported that the yield stress of suspensions of platelike kaolin particles was proportional to the square of the floc concentration at solids concentrations of less than 5% by volume, while at greater concentrations (in the range of the present studies) it increased faster than the square of the floc concentration in a manner consistent with Equation (2*a*).

In dilute flocculated suspensions the floc structure may be considered as immobilizing the water within the individual flocs. Thus α the ratio of immobilized water volume to the solid volume, is an additional parameter characterizing the flocs. From a material balance the density of a floc is given by

$$\rho_f = \frac{\rho_p + \alpha\rho}{1 + \alpha} \qquad (3)$$

The volume fraction solids within the floc is

$$\varphi_{fp} = \frac{1}{1 + \alpha} \qquad (4a)$$

and the volume fraction flocs in a suspension is

$$\varphi_f = (1 + \alpha)\, \varphi \qquad (4b)$$

Floc diameters determined from hindered-settling measurements have been correlated empirically (*9*) with α values obtained simultaneously:

$$\frac{D_f}{D_p} = (1 + \alpha)^2 \qquad (5)$$

As pointed out above a floc does not possess a property exactly analogous to the surface tension. Nevertheless the attractive force between particles plays a somewhat similar role in opposing the elongation of a floc. The magnitude of this pseudo surface tension may be estimated from a floc model similar to the one used previously in describing the rheological and hindered-settling characteristics of flocculated suspensions. For a layer of flocculated particles 1-particle diam. thick and 1 cm. long the floc tension is $\sigma_f = 6F\varphi/\pi D_p$. Relating the attractive force between particles to the yield stress as in the previous paper (*9*) one gets

$$\sigma_f \approx \tau_y D_p/\varphi_{fp}^2 = (1 + \alpha)^2 \tau_y D_p \qquad (6)$$

BEHAVIOR OF FLOCS IN SIMPLE SHEAR

When the rate of shearing strain in parallel flow is small, the motion of a floc is determined by two related factors. Because the floc has non-Newtonian flow characteristics the apparent viscosity may be quite large compared with that of the suspending medium. Under these conditions Taylor's analysis (*1*) for the case of viscous forces in the drop much larger than surface tension forces applies, and the floc may be expected to rotate with little accompanying deformation and a period (*16*) of $4\pi/(du/dr)$.

As the rate of shearing strain is increased, the apparent viscosity of the floc decreases and the deformation is increased. Taylor's analysis for liquid-liquid emulsions (which holds for $\mu_e/\mu_o > 10$) predicts that drop dimensions are a simple function of the viscosity ratio

$$D_L = \frac{L - B}{L + B} = \frac{5}{4(\mu_e/\mu)} \qquad (7)$$

although experimental values were found to be about three times the predicted value. However the details of floc deformation are more complicated than those observed with emulsions because as the floc deforms the relative positions of individual particles in a floc may be altered, producing a more dense structure.

The increase in individual floc density as the rate of shearing strain is increased may be evaluated from the laminar flow shear diagram if it is assumed that the principal effect of the flocculated solids on the energy dissipation in dilute flowing suspensions is due to the distortion of the flow field by the flocculated solids and not due to the rupture of particle-particle bonds within a floc. With this assumption the effective volume fraction of flocs in a suspension φ_{eff} may be calculated from the apparent viscosity of the suspension at a given rate of shearing strain

$$\mu_a = \frac{g_c \tau_w}{du/dr} = \frac{g_c \tau_w}{\left\{ \frac{3}{4} + \frac{1}{4} \left[\frac{d \ln(8V/D)}{d \ln(D\Delta p/4L)} \right] \right\} \frac{8V}{D}} \quad (8)$$

and the curve relating the viscosity of a Newtonian suspension to the true volume fraction solids (*17*) (Figure 1).

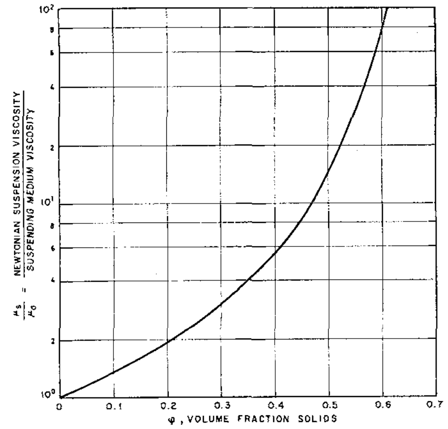

Fig. 1. Effect of suspension concentration on the viscosity of Newtonian suspensions.

Within this framework the effective volume fraction of flocs in a suspension corresponding to the value of η/μ represents a limiting value for φ_{eff} for very high rates of shear and will be designated as φ_∞. The ratio of φ_{eff} to the true volume fraction solids φ_o is by definition $(1 + \alpha)$ which in turn is related to the volume fraction solids and density of individual flocs by Equations (3) and (4). Values of the expression $(\alpha - \alpha_\infty)/\alpha_\infty$ calculated following the above procedures for a wide variety of suspensions and suspension concentrations (14) are shown in Figure 2 as a function of the rate of shearing strain.

Figure 2 also contains data from hindered-settling rate measurements on the same suspensions (9). In this case the velocity gradient at the surface of the individual floc was calculated from Stokes' law. The effective volume fraction solids at high rates of shear was calculated from Figure 1 with η/μ values from Equation (2*b*). The appropriate volume fraction solids for substitution in Equation (2*b*) is that given by the α values determined from the hindered-settling measurements.

The remarkable fact about the data shown in Figure 2 is that despite the range of conditions the data are fitted by a single expression

$$\frac{\alpha - \alpha_\infty}{\alpha_\infty} = k_1 \left(\frac{du}{dr} \right)^{-1/2} \quad (9)$$

where $k_1 = 14$ sec.$^{-1/2}$. At the low shear rates the present data are in rather good agreement with the data for the highest shear rates given by Michaels and Bolger (*8*) for suspensions of platelike kaolin particles.

The effect of the rate of shear in laminar flow on the floc diameter can now be evaluated by combining the results of a material balance with Equations (5) and (9) and assuming a constant number of particles in a floc. The result is

$$\frac{D_f}{D_p} = \left[\frac{\alpha_\infty k_1}{(du/dr)^{1/2}} + (1 + \alpha_\infty) \right]^{1/3} (1 + \alpha)^{5/3} \quad (10)$$

In Equation (10) the α_∞ values are a function of the concentration since they are determined from the value of η/μ for a given suspension concentration, while the α value is characteristic only of the particular material studied because it is determined from dilute suspension hindered-settling studies. Comparison of Equations (9) and (10) shows that although the rate of shear *du/dr* exerts a marked effect on the value of α, the effect on the floc diameter is much smaller. For instance in the limits of very large rates of shear the first term in the brackets on the right goes to zero. Since α_∞ is always less than α, this

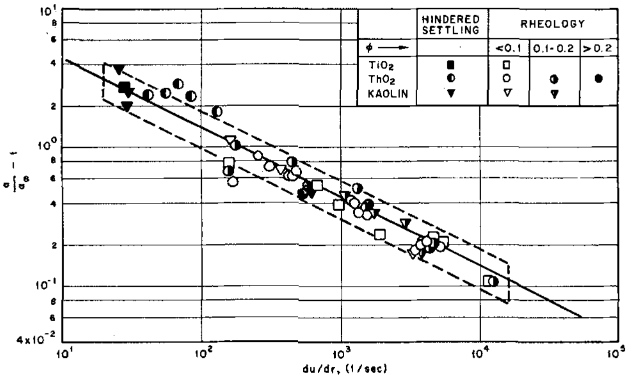

Fig. 2. Effect of velocity gradient in laminar flow on ratio of immobilized water volume to solid volume in individual flocs.

means that the floc diameter is always within the limits

$$(1 + \alpha)^{5/3} < (D_f/D_p) \leqq (1 + \alpha)^2 \quad (11)$$

over the complete range of laminar shear rates on interest. It must be emphasized that Equations (9) to (11) apply only to the third case of deformations in which the apparent viscosity of the floc is sufficient to prevent elongation of the floc under the action of shear forces $(\mu_a/\mu) > 10$. Evaluation of the viscosity ratio with the data of Figure 2 and Equation (4), (2*a*), (2*b*), and (1*a*) showed that in all cases the above criteria were met.

FLOC DISRUPTION BY TURBULENCE

It is a commonly accepted view that the action of turbulent mixing is to stretch material lines and surfaces (*18*). Although flocs may not be distorted as much as fluid elements because of the structure imparted by particle-particle interactions, some deformation of the floc is to be expected. In general the surface of the floc will be extended in the same manner as predicted by Batchelor (*18*), and the floc will assume the form of a filament or sheet.

Oscillations of the floc will not play as important a role in floc breakup as the rupture of liquid drops. This is because the particle-particle interactions tend only to oppose floc rupture and do not tend to restore a floc to the condition in which the surface energy is a minimum as does the surface tension in liquid drops (*19*). Consequently the flocs do not possess a characteristic frequency of oscillation, and so resonance mechanisms (*2*) or vortex shedding (*20*) are relatively unimportant in determining the equilibrium floc size.

The primary mechanism leading to disruption of the extended flocs by turbulence is pressure difference on opposite sides of the floc which cause bulgy deformation and eventual rupture. The pressure differences are due to the random velocity fluctuations of turbulent flow. Although by their very randomness it is as yet impossible to describe the turbulent fluctuations exactly, useful results may be obtained by assuming a simplified flow model in which the pipe cross section is divided into two regions: the central portion of the channel where local isotropy may be assumed in the manner proposed by Kolmogoroff (*21, 22*), and the region adjacent to the channel walls where the production of small eddies of great intensity is at a maximum. Such a division has been used by Levich (*4*) in treating the breakup of liquid drops in turbulent flow. The necessity for this somewhat arbitrary division has been shown experimentally by Sleicher (*3*). In that study photographs of the drop breakup process showed that the rupture of drops only slightly greater than the maximum stable drop size occurred in the region very close to the wall of the tube. Despite this observation the data were correlated with dimensional arguments similar to those advanced by Hinze (*2*) and which are expected to apply largely to the central portion of the stream.

A factor opposing the production of small floc sizes by turbulent disruption is the agglomeration of flocs as they are brought into contact by local velocity gradients. The balance between the disruption and agglomeration processes determine the equilibrium distribution of floc sizes.

The effect of floc concentration and turbulence level on the steady state floc size may be obtained by considering the simplest model which allows for both collision and rupture of flocs. Near the steady state distribution the floc size may be increased by the collision of two flocs. The lifetime of this floc is then determined by the magnitude of the interparticle forces and the frequency with which the floc might be expected to encounter a pressure fluctuation larger than the interparticle forces. The variation of the number of flocs N_A is given by

$$\frac{dN_A}{dt} = KN_A^2 - \beta N_B \qquad (12)$$

where K is the rate constant giving the rate of collision of flocs of size D_{fA} to form flocs of size D_{fB}, and β is the rate of rupture of D_{fB} to form D_{fA}. At steady state $dN_A/dt = 0$, and

$$N_A^2 = \frac{\beta}{K} N_B \qquad (13)$$

A material balance gives

$$N_A + 2N_B = \frac{6(1+\alpha)\varphi}{\pi D_{fA}^3} = N_{Ao} \qquad (14)$$

Combining Equations (13) and (14) and solving for N_A, retaining terms through N_{Ao}^2, one gets

$$2K N_{Ao}/\beta = (N_{Ao} - N_A)/N_{Ao} \qquad (15)$$

The value of N_A may be estimated with the assumption that D_{fB} flocs are destroyed virtually as soon as they are formed; that is that $\beta/K >> 1$. Then

$$-\int_{N_{Ao}}^{N_A} \frac{dN_A}{N_A^2} = K \int_o^t dt \qquad (16)$$

and

$$N_A = \frac{N_{Ao}}{Kt\, N_{Ao} + 1} \qquad (17)$$

Substitution of Equation (17) into Equation (15) gives

$$2K N_{Ao}/\beta = Kt\, N_{Ao}/(Kt\, N_{Ao} + 1) \qquad (18)$$

The values of the rate constants are determined by the characteristics of the turbulent flow pattern in the turbulent core and in the wall region. Manley and Mason (*16*) have proved experimentally the theoretical results due to Smoluchowski (*5*) that the rate of collision of particles in laminar flow is

$$K_l = \frac{1}{12} D_{fA}^3 \frac{du}{dr} \qquad (19)$$

while Saffman (*23*) has derived an expression for the rate of collision due to spatial variations of the turbulent velocity:

$$K_t = 1.30\, D_{fA}^3\, (\epsilon/\nu)^{1/2} \qquad (20)$$

These expressions are substantially the same except for the numerical constant, since the term $(\epsilon/\nu)^{1/2}$ is directly proportional to the root-mean-square velocity gradient in isotropic turbulence.

A floc will be disrupted when the dynamic pressure on the opposite sides of the floc exceeds the yield stress. The yield stress is given by Equation (*2a*), provided the volume fraction solids of the floc particles φ_{fp} is used. Values of the dynamic pressure fluctuations are related to the magnitude of the turbulent velocity fluctuations by

$$p' \approx 0.7\, \overline{\rho\, u'^2}/g_c \qquad (21)$$

in isotropic turbulence and by

$$p' \approx 0.005\, \overline{\rho\, U^2_{\max}}/g_c \approx \overline{2\, \rho\, u'^2}/g_c \qquad (22)$$

at the wall in a turbulent boundary layer (*24*). A force balance gives

$$u' = \sqrt{\frac{g_c\, \tau_y}{k_2\, \rho}} \qquad (23)$$

where $k_2 \approx 0.7$ for isotropic turbulence and $k_2 \approx 2$ for the wall region in a turbulent boundary layer.

The rate of floc rupture β is somewhat more difficult to arrive at than the value of K, but it is approximately given by the frequency of occurrence of velocity fluctuations equal to or greater than the value given by Equation (23). At this point it is necessary to divide the discussion into a section on the characteristics of flow in the turbulent core and flow in the region of a surface, the value of the small-scale velocity fluctuations of Equation (23) being the particular parameter sought.

Turbulent Core Region

Corrsin (*25*) has postulated that the Kolmogoroff theory of local isotropy (*22, 26*) may be applied to situations involving turbulent shear flow provided the local transfer time is shorter than the characteristic time of the gross shear strain, or that $(\epsilon/\nu)^{1/2} >> \partial u/\partial y$. This criterion is believed to be fulfilled for the central portion of flow in a channel provided the Reynolds number is sufficiently large. On this basis the small-scale velocity fluctuations are essentially independent of the main flow and are determined by the rate of local energy dissipation ϵ and the kinematic viscosity ν. Kolmogoroff postulated that under these conditions the flow in the universal equilibrium range could be characterized by a length scale

$$\gamma = (\nu^3/\epsilon)^{1/4} \qquad (24)$$

a velocity scale

$$v = (\nu\epsilon)^{1/4} \qquad (25)$$

and a time scale

$$\theta = (\nu/\epsilon)^{1/2} \qquad (26)$$

In the universal equilibrium range the distribution of the intensity of turbulence over the range of wavenumbers (or over the range of eddy sizes) is given by

$$\overline{[u'(r)]^2} = \int_k^\infty E(k, t)\, dk \tag{27}$$

where

$$E(k, t) = C_2\, \epsilon^{2/3}\, k^{-5/3} \text{ for } L >> r >> \eta \tag{28}$$

and

$$E(k, t) = C_3\, \epsilon / k^3\, \nu \text{ for } L >> \eta >> r \tag{29}$$

The length L is the linear scale of the energy containing eddies $C_2 \approx 0.73$ in accordance with Hinze (*22*) and $C_3 \approx 4/15$ in accordance with Obukhoff (*27*).

The frequency of eddies (that is the rate of disruption of flocs) is given approximately by (*22*)

$$\beta \approx u'k \tag{30}$$

Integrating Equation (27) and equating the result to Equation (23) one obtains the wave number in terms of the local energy dissipation and the magnitude of the fluctuating velocity required to rupture a floc:

$$k = 0.66\, \epsilon / (g_c\, \tau_y / \rho)^{3/2} \text{ for } L >> r >> \eta \tag{31}$$

and

$$k = 0.30\, (\rho \epsilon / g_c\, \tau_y\, \nu)^{1/2} \text{ for } L >> \eta >> r \tag{32}$$

In order to effectively disrupt the floc the mean-square relative velocity $\overline{u'(r)^2}$ must relate to a distance of the order of the floc diameter; the required relation is (*22, 27*)

$$\overline{[u'(r)]^2} = \frac{3}{2} C_2\, \epsilon^{2/3}\, r^{2/3} \text{ for } L >> r >> \eta \tag{33}$$

and

$$\overline{[u'(r)]^2} = \frac{1}{2} C_3\, \epsilon\, r^2 / \nu \text{ for } L >> \eta >> r \tag{34}$$

Substitution of Equations (31) through (34) into Equation (30) gives the required rate constant for use in Equation (18). The one remaining term to be evaluated is the value of the time in this equation. In the derivation of Equation (18) it was assumed that near equilibrium the large flocs formed by collision were disrupted virtually as soon as they were formed. This suggests that a suitable value of the time can be estimated from the average distance separating flocs divided by the characteristic Kolmogoroff velocity, Equation (25). For flocs arranged in a cubic array the value of the time is given by

$$t = \frac{D_f \left[\left(\dfrac{0.524}{(1+\alpha)\varphi} \right)^{1/3} - 1 \right]}{(\nu\, \epsilon)^{1/4}} \tag{35}$$

Replacing the term r in Equations (33) and (34) by D_f and collecting the appropriate values for substitution in Equation (18) one obtains the required relations for steady state floc size in the region where the assumption of local isotropy is valid:

$$D_f = C_4 (g_c\, \tau_y / \rho)^{9/2}\, \nu^{-3/2}\, \epsilon^{-5/2}\, [1 + C_5 (1 + \alpha)\, \varphi]^3 \text{ for } L >> r >> \eta \tag{36}$$

and

$$D_f = C_6 (g_c\, \tau_y / \rho)^{1/2}\, (\nu / \epsilon)^{1/2}\, [1 + C_5 (1 + \alpha)\, \varphi] \text{ for } L >> \eta >> r \tag{37}$$

where $0.7 < C_4 < 125$ with a most probable value of 25; $10 < C_6 < 30$ with a most probable value of 19, and

$$C_5 = \frac{C_7\, D_f \left[\left(\dfrac{0.524}{(1+\alpha)\,\varphi} \right)^{1/3} - 1 \right]}{(\nu^3 / \epsilon)^{1/4}} \tag{38}$$

where $0.75 < C_7 < 25$ with a most probable value of 2.5.

Wall Regions

Near the wall where the average scale of the primary motion is small (that is $y^+ < 10$), there is a strong resonance between the primary motion and the turbulent motion. In this region the spectral law is (*28*)

$$E(k, t) = \frac{\epsilon_w}{\alpha''\, k (du/dr)} \tag{39}$$

In the region $10 < y^+ < 30$ the average scale of the primary motion is large, and the resonance between the primary motion and the turbulent motion is not as important as indicated by Equation (39). In this region the transfer of energy among the eddies predominates, and the spectral law is the $k^{-5/3}$ law given earlier [Equation (28)]. The extensive studies of Laufer (*22*) indicate that the local energy dissipation per unit mass is at a maximum near $y^+ = 12$ with a value of $\epsilon_w D / 2u_*^3 \approx 230$. Since the region of maximum energy dissipation will be the region of minimum floc size, the $k^{-5/3}$ spectral law will be used in further calculations. If it is assumed that the velocity gradient of the mean motion in the wall region is the primary factor influencing floc-floc collisions, then the collision rate is given by Equation (19). Combining Equations (18), (19), (23), (30), (31), (33), and (35) one gets the floc size in the wall region

$$D_f = C_8 (du/dr)^3\, (g_c\, \tau_y / \rho)^{9/2}\, \epsilon_w^{-4}\, [C_5 (1 + \alpha)\, \varphi + 1]^3 \tag{40}$$

where $0.1 < C_8 < 1$ with a most probable value of 0.5.

Distribution of Floc Sizes

The ratio of the floc size in the wall region of a pipe to the floc size in the core for $L >> r >> \eta$ is given by the ratio of Equations (40) and (36):

$$\frac{(D_f)_{\text{wall}}}{(D_f)_{\text{core}}} = \frac{C_8\, (du/dr)^3}{C_4\, (\epsilon/\nu)^{3/2} (\epsilon_w / \epsilon)^4} \tag{41}$$

where the term (ϵ_w/ϵ) is the ratio of maximum to average local energy dissipation per unit mass. For typical conditions the ratio of floc sizes in the two regions is

$$0.3 > [(D_f)_{\text{wall}} / (D_f)_{\text{core}}] > 0.001 \tag{42}$$

For suspensions of micron-sized particles the floc size under low shear (hindered settling) conditions is of the order of 100 μ (*9*). Thus under most circumstances the suspension will be essentially deflocculated in the region immediately adjacent to the tube walls.

The floc size distribution for the combined wall region and turbulent core will be a function of the rate of diffusion of flocs into and out of the two regions and the residence time in the representative regions. However since the wall region represents less than 20% of the flow area under any except extreme flow conditions, the value for the mean floc size will be given quite closely by the expression for the turbulent core, Equation (36).

DISCUSSION OF RESULTS

The essential difference between the mechanisms for breakup of liquid drops and the rupture of suspension flocs is that drop oscillations play a primary role in the breakup of drops in liquid-liquid systems, whereas the turbulent intensity must be sufficiently great to shear the floc structure in solid-liquid suspensions. This means that since drop oscillations may be excited over a rather wide range of turbulent intensities, the drop size is not strongly dependent on the values of the local turbulent energy dissipation. However since the turbulent pressure difference must ex-

Table 1. Comparison of Factors Effecting Drop Breakup and Floc Rupture

Diameter proportional to:	Wall region	Turbulent core
Drop breakup		
Kolmogoroff (*21*)	—	$\sigma^{3/5}\epsilon^{-2/5}$
Hinze (*2*)	—	$\sigma^{3/5}\epsilon^{-2/5}$
Levich (*4*)	$\sigma^{1/2}\epsilon_w^{-2/3}$	—
Sleicher (*3*)	$\sigma^{3/2}\nu^{-1/2}\epsilon^{-25/28}$	—
Floc breakup		
$L >> r >> \eta$	$(du/dr)^3(\sigma_f/D_p)^{9/2}\epsilon_w^{-4}$	$(\sigma_f/D_p)^{9/2}\nu^{-3/2}\epsilon^{-5/2}$
$L >> \eta >> r$	—	$(\sigma_f/D_p)^{1/2}(\nu/\epsilon)^{1/2}$

ceed the yield stress before a floc can be sheared, the floc size will be strongly dependent on the value of the local turbulent energy dissipation, almost to the extent of showing a step function decrease in floc size from the virtual equilibrium size observed in laminar flow [Equation (11)] to the equilibrium floc size for turbulent flow with the primary particle size of the suspension setting a lower limit on the floc size distribution. These two situations are compared qualitatively in Table 1 in which the expressions for drop breakup by Sleicher (*3*), Kolmogoroff (*21*), and Hinze (*2*) for the overall pipe flow, and by Levich (*4*) for breakup in the wall region are presented in terms of the local energy dissipation per unit mass, rather than the mean flow parameters of the original papers. In the expression for the factors affecting the floc size it was assumed that the effective tension resisting floc deformation σ_f is given by Equation (6). It is clear from Table 1 that for the case where $L >> r >> \eta$ (that is the inertial subrange), the floc diameter shows a much stronger dependence on the turbulent energy dissipation than does the drop diameter. Even for the case where viscous dissipation is becoming important ($L >> \eta >> r$), the floc size shows a somewhat stronger dependence on ϵ than does the drop size.

The very strong dependence of floc size on the local energy dissipation, once the turbulent intensity is sufficient to rupture flocs, is confirmed by the data of Reich and Vold (*7*) obtained by agitation of flocculated ferric oxide suspensions. Figure 3 shows their data for different suspension concentrations plotted as floc diameter in microns vs. the local energy dissipation in arbitrary units. The local energy disspiation was calculated on the assumption that (*29*)

$$\epsilon = K_1N^3D^2 \tag{43}$$

where the length term refers to the impeller diameter, a constant during the above studies. Apparently the method of preparation resulted in an equilibrium floc size of about 100 μ at low rates of shear. This size is consistent with other observations based on hindered-settling studies (*9*). As the turbulent energy dissipation was increased up to a critical value, which was dependent on concentration, there was very little decrease in floc size. Beyond this critical value of the energy dissipation the floc size decreased very rapidly with just the $\epsilon^{-5/2}$ dependence predicted by Equation (36) until a limiting value of the order of 1 to 4 μ was reached.

Additional experimental confirmation of the form of Equation (36) may be obtained by plotting Reich and Vold's data as floc size vs. volume fraction solids, Figure 4. Equation (36) predicts that for $[C_5(1 + \alpha)\varphi] >> 1$ the floc size should be proportional to the cube of the volume fraction solids. This is just the trend shown by the data for the range of floc sizes between the limiting values of 1 to 4 μ and 100 μ observed in the previous figure.

Although a quantitative check of the constants in the theoretical expressions cannot be obtained directly from Reich and Vold's data, a qualitative estimate can be made. Since no values for suspension properties or energy dissipation per unit mass were given, it will be necessary to estimate them from other sources. Previous studies (*9*) of suspensions have shown that

$$\tau_y/\varphi^3 = k_3\,\alpha^4 \tag{44}$$

The correct value for φ in Equation (44) is that for the volume fraction solids in the floc, or simply $1/(1 + \alpha)$ from Equation (4*a*). For $\alpha >> 1$ Equation (44) becomes

$$\tau_y = k_3\,\alpha \tag{45}$$

where $k_3 = 1.55 \times 10^{-2}$ lb.$_f$/sq. ft. A reasonable value

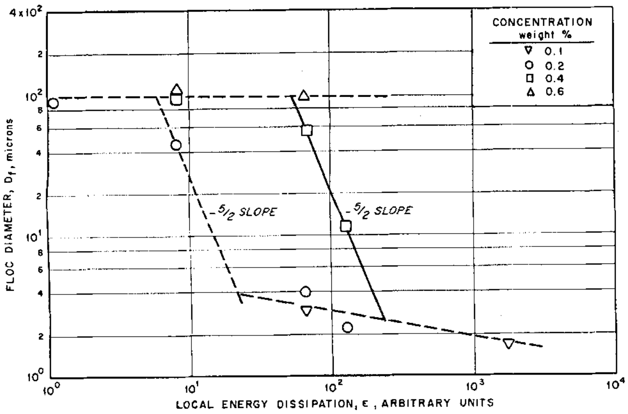

Fig. 3. Effect of local energy dissipation on floc size in ferric oxide suspensions (data of Reich and Vold).

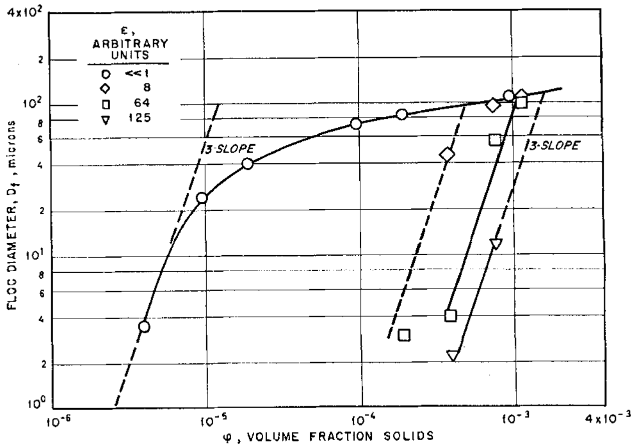

Fig. 4. Effect of volume fraction solids on floc size of ferric oxide suspensions (data of Reich and Vold).

for α in highly flocculated suspensions is 20. Based on the dimensions of the agitator and vessel and the extensive data given by Rushton (*30*) the particular value of K_1 in Equation (43) for Reich and Vold's system may be estimated as being between 0.5 and 1.5. Using these values in Equation (36) one obtains a value of C_4 in the range of 1 to 5. This is in quite good agreement with the range $(0.7 < C_4 < 125)$ predicted from the theoretical analysis in view of the approximations involved.

An estimate of the floc size in pipe flow may be obtained from the appropriate equations for floc size [Equations (36), (37), or (40)] and an approximate expression for the mean rate of energy dissipation per unit mass in pipe flow (*31*)

$$\epsilon = \frac{g_c\,\Delta p}{\rho L} V = 2f\frac{V^3}{D} \tag{46}$$

which assumes that the core of the flow is nearly homogeneous and dissipates most of the energy. In the vicinity of the wall the energy dissipation may be approximated by (32)

$$\epsilon = \frac{g_c\,\tau}{\rho}\frac{du}{dr} \tag{47}$$

with a maximum of the order of $\epsilon D/2u_*^3 \approx 230$, based on Laufer's data (22).

NOTATION

B = breadth, ft.
$C_{1 \text{ to } 8}$ = constants, dimensionless
D = diameter, ft.
$E(k, t)$ = energy spectrum function, cu. ft./sec.^{2}
g_c = conversion factor, (lb.$_m$/lb.$_f$) (ft./sec.^{2})
k = wave number, ft.$^{-1}$
k_1 = constant, sec.$^{-1/2}$
$k_2\,K_1$ = constant, dimensionless
k_3 = constant, lb.$_f$/sq. ft.
K = rate constant, cu. ft./sec.
K_l = collision rate in laminar flow, cu. ft./sec.
K_t = collision rate due to turbulent fluctuations, cu. ft./sec.
L = length, dimensionless
N = particles per unit volume, ft.$^{-3}$
p' = fluctuating pressure, lb.$_f$/sq. ft.
Δp = pressure loss, lb.$_f$/sq. ft.
r = length, ft.
t = time, sec.
u_* = friction velocity = $(g_c\,\tau/\rho)^{1/2}$, ft./sec.
u' = fluctuating velocity, ft./sec.
$U_{\max}$ = maximum stream velocity, ft./sec.
du/dr = velocity gradient, sec.$^{-1}$
v = Kolmogoroff velocity, ft./sec.
V = mean velocity, ft./sec.
y = distance from wall, ft.
$>$ = greater than
$>>$ = much greater than

Greek Letters

α = volume immobilized water/volume solid, dimensionless
α'' = numerical constant, dimensionless
β = rate constant, sec.$^{-1}$
γ = Kolmogoroff length, ft.
ϵ = local energy dissipation per unit mass, sq. ft./sec.^{3}
η = coefficient of rigidity, lb.$_m$/ft. sec.
θ = Kolmogoroff time, sec.
μ = viscosity, lb.$_m$/ft. sec.
μ_a = apparent viscosity, lb.$_m$/ft. sec.
μ_e = effective viscosity, lb.$_m$/ft. sec.
ν = kinematic viscosity, sq. ft./sec.
π = 3.14159. . . . . .
ρ = density, lb.$_m$/cu. ft.
σ = surface tension, lb.$_f$/ft.
τ = shear stress, lb.$_f$/sq. ft.
τ_y = yield stress, lb.$_f$/sq. ft.
φ = volume fraction solids, dimensionless

Subscripts

A, B = floc size A and B, $B < A$
f = floc
fp = floc particle
o = zero time
p = particle
w = wall

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